2014 Summer Review for Students Entering Algebra 2

1. Solving Linear Equations
2. Solving Linear Systems of Equations
3. Multiplying Polynomials and Solving Quadratics
4. Writing the Equation of a Line
5. Laws of Exponents and Scientific Notation
6. Solving with Trig Ratios and Pythagorean Theorem

TI-84 Plus Graphing Calculator is required for this course.

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TOPIC 1: Solving Linear Equations

Equations in one variable with fractional coefficients may be solved in a variety of ways. Commonly, the first step is to multiply all the terms by a common denominator to remove the fractions. Then solve in the usual way. Combine like terms. Isolate the variable on one side and the constants on the other. Finally, divide to find the value of the variable.

Example 1: Solve $\frac{1}{2}x + x - 3 = \frac{1}{3}x + 4$

Solution:

	$\frac{1}{2}x + x - 3 = \frac{1}{3}x + 4$	problem
	$6(\frac{1}{2}x + x - 3) = 6(\frac{1}{3}x + 4)$	multiply by the common denominator
	$3x + 6x - 18 = 2x + 24$	simplify
	$9x - 18 = 2x + 24$	simplify
	$7x = 42$	add 18, subtract $2x$ from each side
	$x = 6$	divide

Example 2: Solve for y: $\frac{y}{2} + \frac{y}{3} - 3 = y$

Solution:

	$\frac{y}{2} + \frac{y}{3} - 3 = y$	problem
	$6(\frac{y}{2}) + 6(\frac{y}{3}) + 6(-3) = 6(y)$	multiply by the common denominator
	$3y + 2y - 18 = 6y$	simplify
	$-18 = y$	subtract $5y$ from each side

Solve the following equations.

1. $\frac{2}{3}x - 7 = \frac{1}{3}x + 3$
2. $\frac{5}{4}x - 7 = x - 5$
3. $\frac{1}{3}y = \frac{1}{4}y + 7$
4. $\frac{2}{3}x + 7 = -\frac{1}{5}x + 3$
5. $\frac{1}{3}y + 7 + \frac{1}{6}y = \frac{1}{2}y - 1$
6. $\frac{x}{5} + \frac{2x}{3} + x = 2$

Answers: 1. $x = 30$ 2. $x = 8$ 3. $y = 84$ 4. $x = -\frac{60}{13}$ 5. no solution 6. $x = \frac{15}{14}$

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TOPIC 2: Solving Linear Systems of Equations

When two equations are both in $y = mx + b$ form it is convenient to use the Equal Values Method to solve for the point of intersection. Set the two equations equal to each other to create an equation in one variable and solve for x. Then use the x-value in either equation to solve for y.

If one of the equations has a variable by itself on one side of the equation, then that expression can replace the variable in the second equation. This again creates an equation with only one variable. This is called the Substitution Method. See Example 1 below.

If both equations are in standard form (that is $ax + by = c$), then adding or subtracting the equations may eliminate one of the variables. Sometimes it is necessary to multiply before adding or subtracting so that the coefficients are the same or opposite. This is called the Elimination Method. See Example 2 below.

Sometimes the equations are not convenient for substitution or elimination. In that case one of both of the equations will need to be rearranged into a form suitable for the previously mentioned methods.

Example 1: Solve the following system. $4x + y = 8$

$$x = 5 - y$$

Solution: Since x is alone in the second equation, substitute $5 - y$ in the first equation, then solve as usual.

$$\begin{aligned} 4(5-y)+y &= 8 \\ 20-4y+y &= 8 \\ 20-3y &= 8 \\ -3y &= -12 \\ y &= 4 \\ x &= 5-4 \\ x &= 1 \end{aligned}$$

Then substitute $y = 4$ into either original equation to find x. Using the second equation $x = 1$ so the solution is (1, 4)

$$x = 5 - 4$$

$$x = 1$$

Example 2: Solve the following system. $-2x + y = -7$

$$3x - 4y = 8$$

Solution: If we add or subtract the two equations no variable is eliminated. Notice, however, that if everything in the top equation is multiplied by 4, then when the two equations are added together, the y-terms are eliminated.

$$\begin{aligned}-2x+y&=-7\\3x-4y&=8\end{aligned} \quad\Rightarrow\quad \begin{aligned}4(-2x+y&=-7)\\3x-4y&=8\end{aligned} \quad\Rightarrow\quad \begin{aligned}-8x+4y&=-28\\ \underline{3x-4y}&\underline{=8}\\-5x+0&=-20\\x&=4\end{aligned}$$

Substituting $x = 4$ into the first equation $-2(4) + y = -7 \Rightarrow -8 + y = -7 \Rightarrow y = 1$.
The solution is (4,1).

Solve the following systems of equations.

1. $y = 7x - 5$
   $2x + y = 13$

2. $x + y = -4$
   $-x + 2y = 13$

3. $y = -3x$
   $4x + y = 2$

4. $y - x = 4$
   $2y + x = 8$

5. $2x - y = 4$
   $\frac{1}{2}x + y = 1$

6. $-4x + 6y = -20$
   $2x - 3y = 10$

Answers:

1. (2, 9) 2. (−7, 3) 3. (2, −6) 4. (0, 4) 5. (2, 0) 6. infinite solutions

TOPIC 3: Multiplying Polynomials and Solving Quadratics

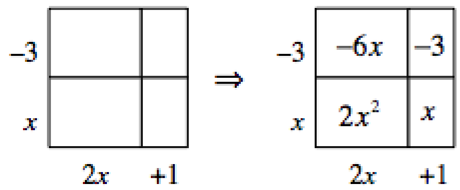
Polynomials can be multiplied (changed from the area written as a product to the area written as a sum) by using the Distributive Property or generic rectangles.

Example 1: Multiply $-5x(-2x + y)$.

Solution: Using the Distributive Property $\underbrace{-5x(-2x+y)}_{\text{area as a product}} = -5x \bullet -2x + -5x \bullet y = \underbrace{10x^2 - 5xy}_{\text{area as a sum}}$

Example 2: Multiply $(x - 3)(2x + 1)$.

Solution: Although the Distributive Property may be used, for this problem and other more complicated ones, it is beneficial to use a generic rectangle to find all the parts.



$\Rightarrow (x - 3)(2x + 1) = 2x^2 - 5x - 3$
area as a product   area as a sum

Solving quadratics first required factoring the polynomials to change the sum into a product. It is the reverse of multiplying polynomials and using a generic rectangle is helpful. Once the expression is factored, then the factors can be found using the Zero Product Property.

Example 3: Solve $x^2 + 7x + 12 = 0$

Solution: First, factor the polynomial. Sketch a generic rectangle with 4 sections.

Write the x^2 and the 12 along one diagonal.

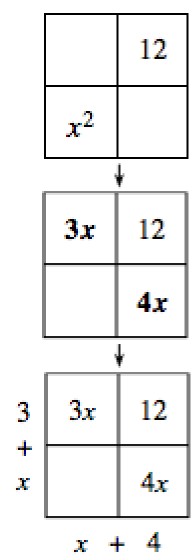
Find two terms whose product is $12 \bullet x^2 = 12x^2$ and whose sum is $7x$. That is, $3x$ and $4x$. This is the same as a Diamond Problem.

Write these terms as the other diagonal.

Find the base and height of the rectangle by using the partial areas.

Write the factored equation. $x^2 + 7x + 12 = (x + 3)(x + 4) = 0$

Then, using the Zero Product Property, we know that either $(x + 3)$ or $(x + 4)$ is equal to zero (since their product is zero). This means



that $x = -3$ or $x = -4$.

Here are some more to try. Multiply the expression in problems 1 through 4 and solve the equation in problems 6 and 7.

1. $2x(x - 1)$ 2. $(3x + 2)(2x + 7)$ 3. $(2x - 1)(3x + 1)$

4. $(x + y)(x + 2)$ 5. $x^2 + 5x + 6 = 0$ 6. $2x^2 + 5x + 3 = 0$

1. $2x^2 - 2x$ 2. $6x^2 + 25x + 14$ 3. $6x^2 - x - 1$

4. $x^2 + xy + 2x + 2y$ 5. $x = -2, -3$ 6. $x = -\frac{3}{2}, -1$

TOPIC 4: Writing the Equation of a Line

Except for a vertical line, any line may be written in the form $y = mx + b$ where *"b"* represents the y-intercept of the line and *"m"* represents the slope. (Vertical lines are always of the form $x = k$.) The slope is a ratio indicating the steepness and direction of the line. The slope is calculated by $m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$.

Example 1: Write the equation of the line with slope $-\frac{1}{2}$ and passing through the point (6, 3).

Solution: Write the general equation of a line. $y = mx + b$

Substitute the values we know for m, x, and y. $3 = -\frac{1}{2}(6) + b$

Solve for b.

$3 = -3 + b$

$6 = b$

Write the complete equation.

$y = -\frac{1}{2}x + 6$

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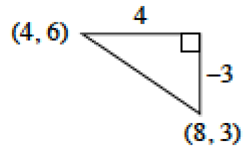
Example 2: Write the equation of the line passing through the points (8, 3) and (4, 6).

Solution: Draw a generic slope triangle

Calculate the slope using the given two points.



$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$,

$m = -\frac{3}{4}$

Write the general equation of a line.

$y = mx + b$

Substitute m and one of the points for *x* and *y*, in this case (8,3).

$3 = -\frac{3}{4}(8) + b$

Solve for *b*.

$3 = -6 + b$

$9 = b$

Write the complete equation.

$y = -\frac{3}{4}x + 9$

Use the given information to find an equation of the line.

1. slope = 5, through (3, 13)

2. slope = $-\frac{5}{3}$, through (3, −1)

3. slope = $\frac{1}{3}$, through (6, 9)

4. through (1, 3), (−5, −15)

5. through (2, −1), (3, −3)

6. through (1, −4), (−2, 5)

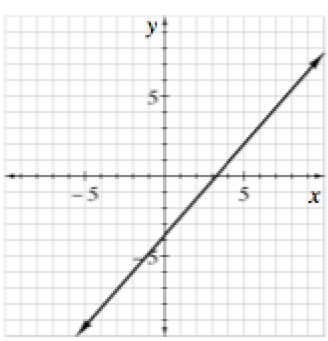
7.

x	−2	−1	0	1	2
y	−3	−1	1	3	5

8.

x	−4	−2	0	2	4
y	1	2	3	4	5

9.



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Answers:

1. y = $5x - 2$
2. $y = -\frac{5}{3}x + 4$
3. $y = \frac{1}{3}x + 7$
4. $y = 3x$
5. $y = -2x + 3$
6. $y = -3x - 1$
7. $y = 2x + 1$
8. $y = \frac{1}{2}x + 3$
9. $y \approx \frac{6}{5}x - 4$

TOPIC 5: Laws of Exponents and Scientific Notation

The laws of exponents summarize several rules for simplifying expressions that have exponents. The rules are true for any base if $x \neq 0$.

$x^a \cdot x^b = x^{(a+b)}$

$(x^a)^b = x^{ab}$

$\frac{x^a}{x^b} = x^{(a-b)}$

$x^{-a} = \frac{1}{x^a}$

$x^0 = 1$

Scientific notation is a way of writing a number as a product of two factors separated by a multiplication sign. The first factor must be less than 10 and greater than or equal to 1. The second factor has a base of 10 and an integer exponent.

Example 1: Simplify $4^2 \cdot 4^{-4}$

Solution: In a multiplication problem, if the bases are the same, add the exponents and keep the base. If the answer ends with a negative exponent, take the reciprocal and change the exponent to positive.

$4^2 \cdot 4^{-4} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

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Example 2: Simplify $\frac{(x^2)^3 \cdot y^4}{x^{-2} \cdot y}$

Solution: Separate the fraction into two fractions with bases x and y. With an exponent on an exponent, multiply the exponents. Next, to divide expressions with exponents and the same base, subtract the exponents.

$$\frac{(x^2)^3 \cdot y^4}{x^{-2} \cdot y} = \frac{(x^2)^3}{x^{-2}} \cdot \frac{y^4}{y} = \frac{x^6}{x^{-2}} \cdot \frac{y^4}{y^1} = x^8 \cdot y^3 = x^8 y^3$$

Example 3: Multiply and give the answer in scientific notation. $(8 \times 10^4) \cdot (4.5 \times 10^{-2})$

Solution: Separate the number parts and the exponent parts. Multiply the number parts normally and the exponent part by adding the exponents. If this answer is not in scientific notation, change it appropriately.

$(8 \times 10^4) \cdot (4.5 \times 10^{-2}) = 8 \times 4.5) \cdot (10^4 \times 10^{-2}) = 36 \times 10^2 = (3.6 \times 10^1) \times 10^2 = 3.6 \times 10^3$

Simplify each expression. For problems 7 and 8 write the final answer using scientific notation.

1. $3^3 \cdot 3^3 \cdot 3^6$
2. $5^4 \cdot 5^{-1}$
3. $x^2 \cdot (x^4)^{-2}$
4. $\frac{(y^2)^3}{y^6} \cdot y^4$
5. $(7^2 \cdot 7^3)^4$
6. $\frac{y^3 \cdot y^2 \cdot y^{-3}}{y^{-4} \cdot y^3}$
7. $(4.25 \times 10^3) \cdot (2 \times 10^5)$
8. $(1.2 \times 10^4) \cdot (7.1 \times 10^{-2})$

Answers:

1. 3^{12}
2. 5^3
3. $x^{-6} = \frac{1}{x^6}$
4. y^4
5. 7^{20}
6. y^3
7. 8.5×10^8
8. 8.52×10^2

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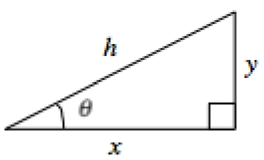
TOPIC 6: Solving With Trigonometric Ratios and the Pythagorean Theorem

Three trigonometric ratios and the Pythagorean Theorem can used to solve for the missing side lengths and angle measurements in any right triangle.

In the triangle below, when the sides are described relative to the angle θ, the opposite leg is y and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.



$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{h}$$

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{h}$$

Also for the triangle above, from the Pythagorean Theorem: $h^2 = x^2 + y^2$.

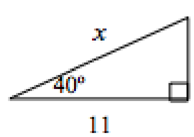
Example 1: A rectangle has a diagonal of 16 cm and one side of 11 cm. What is the length of the other side?

Solution: The diagonal represents the hypotenuse of a right triangle and the one given side represents one of the legs. Using the Pythagorean Theorem:

$$h^2 = x^2 + y^2 \Rightarrow 16^2 = 11^2 + y^2 \Rightarrow 256 = 121 + y^2$$

$$135 = y^2 \Rightarrow y = \sqrt{135} \approx 11.62 \text{ cm}$$

Example 2: Solve for x in the triangle at right.

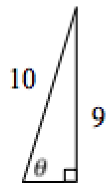


Solution: Based on the 40º angle, 11 is the adjacent side and x is the hypotenuse. Use the cosine

ratio to solve.

$$\cos 40^\circ = \frac{11}{x}$$
$$x\cos 40^\circ = 11 \Rightarrow x = \frac{11}{\cos 40^\circ} \approx 14.36 \text{ units}$$

Example 3: A ten-foot ladder is leaning against the side of a house. If the top of the ladder touches the house nine feet above the ground, what is the angle made by the ladder and the ground?

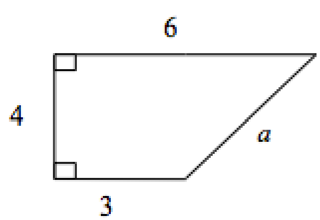


Solution: Make a diagram of the situation similar to the one at right. The ladder (10) is the hypotenuse and the house (9) is the opposite leg. Using the sine ratio, $\sin\theta = \frac{9}{10}$ To find θ, "undo" the sine function with the inverse sine function ($\sin^{-1} x$) as follows:

$$\sin\theta = \tfrac{9}{10}$$
$$\sin^{-1}(\sin\theta) = \sin^{-1}(\tfrac{9}{10})$$
$$\theta = \sin^{-1}\tfrac{9}{10}$$
$$\theta \approx 64.2^\circ$$

Here are some more to try. Use the right triangle trigonometric ratios or the Pythagorean Theorem to solve for the variable(s).

1.



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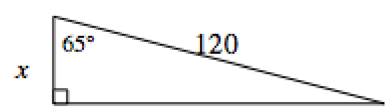

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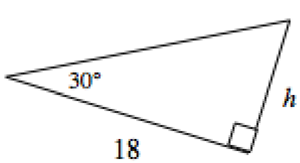

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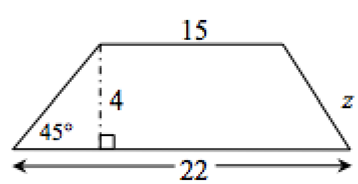

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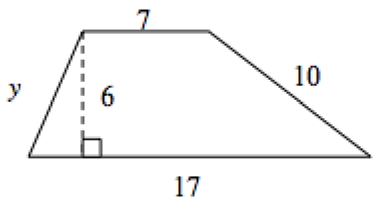

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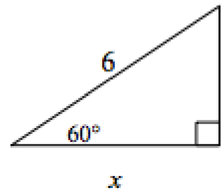

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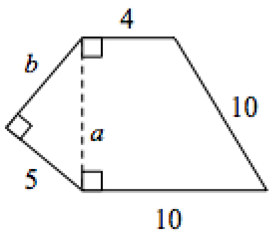

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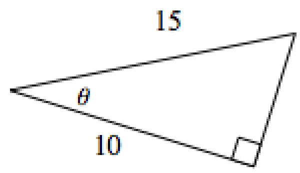

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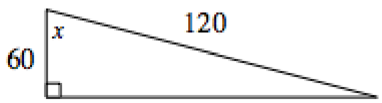

Draw a diagram and solve the following.

10. What is the distance between (−6, −6) and (−3, 2)?

Answers:

1. 5 units
2. ≈ 50.7 units
3. ≈ 10.4 units
4. 5 units
5. $\sqrt{40} \approx 6.32$ units
6. 3 units
7. $a = 8,\ b = \sqrt{39} \approx 6.24$ units
8. ≈ 48.2°
9. 60°
10. $\sqrt{73} \approx 8.5$ units

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